Solution of Math Placement Teat Practice Problems.

Algebraic Solving

1. $(2a^3b)^{-3} = (2^{-3} \cdot a^{(3\cdot -3)} \cdot b^{-3}) = 2^{-3}a^{-9}b^{-3} = \frac{1}{2^3a^9b^3} = \frac{1}{8a^9b^3}$

2. $(-4x^2y{-}3)^{-2} = \frac{y^6}{16x^4}$

Radical

3. $\sqrt{\frac{1}{64}} = \frac{\sqrt{1}}{\sqrt{64}} = \frac{1}{8}$

4. $\sqrt[3]{-27} = -3$

Complex Numbers

5. Find x and y such that the equation is a true statement.

   $3x + 5yc = 15 + 5c$

   $3x = 15$ and $5yc = 5c$

   $x = 5$ and $y = 1$

6. Find x and y such that the equation is a true statement.

   $10x - 4yc = 20 + 1/2x$

   $x = 2$ and $y = -1/8$

Radicals and Complex Numbers

7. Simplify $\sqrt{27 - 18}$

   $\sqrt{27 - 18} = \sqrt{9} = 3$

8. Simplify $\sqrt{\frac{-1}{25}}$

   $\sqrt{\frac{-1}{25}} = \frac{i}{5}$

   Complex Numbers and Exponents

9. Solve i^{15}

   $i^{15} = (i^3)^5 = -i^5 = -(i^3 \cdot i^2) = -(-i \cdot -1) = -i$

10. Solve i^{47}

    $i^{47} = -i$

    Solving Quadratic Equations by Factoring

11. Solve $3x^2 + 8x + 4 = 0$

    $3x^2 + 8x + 4 = 0 => (3x + 2)(x + 2) = 0$

    Thus, $3x + 2 = 0$ *or* $x + 2 = 0$

    $x = -2/3$ *or* $x = -2$

12. Solve $x^2 + 4 = 0$

    $x = \pm 2$

    Solving Quadratic Equations that are irreducible

13. $x^2 + 7x + 3 = 0$

    $x = \frac{-7 \pm \sqrt{49 - 4(1)(3)}}{2 \cdot 1}$

    $x = \frac{-7 + \pm \sqrt{37}}{2}$

14. $2x^2 + 10x - 1 = 0$

    $x = \frac{-10+\pm\sqrt{108}}{4}$

    Solving and Graphing in Equlities

15. Solve and graph $2 - 4x > 10$

    $2 - 4x > 10 \Rightarrow -4x > 8 \Rightarrow x < -2$

16. $x^2 - 2 \geq 1$

    $x \geq \sqrt{3}$ or $x \leq -\sqrt{3}$

17. Solve $3x + 2y = 5$, $7x - y = 1$

    2nd Eq: $7x - y = 1$, $-y = 1 - 7x$, $y = 7x - 1$

    Into 1st Eq: $3x + 2(7x - 1) = 5$, $17x - 2 = 5$

    $x = \frac{7}{17}$

    Thus $y = 7(7/17) - 1$, $y = 49/17 - 1$

    $y = \frac{32}{17}$

18. Solve $w + 2z = 10$, $4w + z = 5$

    $w = 0$, $z = 5$

    Graphing

19. Sketch the function $f(x) = x^2 + 2$ and identify x and y intercept

    y intercept when $x = 0 : f(0) = 0^2 + 2 = 2$, y intercept at (0,2)

    x intercept when $y = 0 : f(x) = 0$ never occur, No x intercept

20. Sketch the functions $f(x) = 3x - 9$ and identify x and y intercept

    y intercept: (3,0)

    x intercept: (0,-9)

    Decimals

21. Solve $.3x + .2 = .5$

    $.3x = .3 \Rightarrow x = 1$

22. Solve $.5x + .4 = 1.2$

    $x = 1.6$

23. Solve for x: $\frac{x+3}{x-9} = 0$

    Numerator must be zero.

    $(x-9)\frac{x+3}{x-9} = 0 \times (x-9)$

    $x + 3 = 3 \Rightarrow x = -3$

24. Slove for x: $\frac{x-2}{2x+1} = 3$

    $x = -1$

25. $f(x) = 10x - 5$ and find a value x^* such that $f(x^*)$=0

    $10x^* - 5 = 0 \Rightarrow 10x^* = 5 \Rightarrow x^* = 0.5$

26. $f(x) = \alpha x + 2\beta$ where α and β are constants. Find x^* such that $f(x^*) = 0$

    $\alpha x^* + 2\beta = 0 \Rightarrow \alpha x^* = -2\beta \Rightarrow x^* = \frac{-2\beta}{\alpha}$

27. the slope of the equation $= \frac{y_1 - y_0}{x_1 - x_0} = \frac{6-4}{2-1} = 2$

    $y = (slope) * x$+(y-intercept)

$y_1 = (2) * x_1$+(y-intercept)

$6 = (2) * 2$+(y-intercept)

$6 = 4$+(y-intercept)

y-intercept=2

the equation: $y = 2x + 2$

28. Is y=sinx one to one?

29. Is y=x+2 onto?

30. For a right triangle ABC with a=3 and c=5, where c is the hypotenuse, find b.

$a^2 + b^2 = c^2 \Rightarrow 3^2 + b^2 = 5^2 \Rightarrow 9 + b^2 = 25 \Rightarrow b^2 = 16$

$b = 4$

31. For a right triangle ABC with a=13 and b=14, find the hypotenuse c.

$c = 15$

32. Write in interval notation $x \in \mathbf{R} : 10 < x < 20$ where $\mathbf{R}$ denotes the set of all real numbers.
$x \in (10, 20)$

33. Write $x \in (-\infty, 0]$ as an inequility.
$x \leq (0)$

Algebrac solving

34. $(\sqrt{x} - 3\sqrt{y}) \cdot (\sqrt{x} + \sqrt{9y})$

$= (\sqrt{x}\sqrt{x}) + (\sqrt{x}\sqrt{9y}) + (-3\sqrt{y}\sqrt{x}) + (-3\sqrt{y}\sqrt{9y})$

$= (x) + (\sqrt{9xy} + (-3\sqrt{yx}) + (-9y)$

$= x + 3\sqrt{xy} - 3\sqrt{xy} - 9y$

$= x - 9y$

35. $(\sqrt[3]{x} + 2)(x^2 - 4) = x^{7/3} - 4\sqrt[3]{x} + 2x^2 - 8$

36. $\frac{3}{x-1} + \frac{x}{x+4} = (\frac{x+4}{x+4})(\frac{3}{x-1}) + \frac{x-1}{x-1}(\frac{x}{x+4})$

$= \frac{3x+12}{(x+4)(x-1)} + \frac{x^2-x}{(x+4)(x-1)} = \frac{x^2+2x+12}{(x+4)(x-1)} = \frac{x^2+2x+12}{x^2+3x-4}$

37. $\frac{2}{x-2} + \frac{x}{x+3} = \frac{x^2+6}{x^2+x-6}$

38. Convert $320\,yards/hr = \frac{320\,yards}{1\,hour} = \frac{960\,feet}{60\,mins} = 16 ft/min$

39. Convert $r\ feet/sec = \frac{12\,inches}{1/60\,min} = 720\,inches/min$

40. Convert $\frac{5}{12} = 0.4166 = 42\,percentage.$

41. Convert $\frac{9}{13} = 0.6923 = 69\,percentage.$

42. $y = x + 4$ and $y = 3x + 1$

$x + 4 = 3x + 1 \Rightarrow 2x = 3 \Rightarrow x = 3/2$

$y = (3/2) + 4 = 5.5$

$x = 1.5$ and $y = 5.5$

43. $x + y = 12$ and $y - x = 4$

$x = 12 - y \Rightarrow y - (12 - y) = 4 = y - 12 + y = 2y - 12 \Rightarrow 2y = 16 \Rightarrow y = 8$

$y - x = 4 = 8 - x \Rightarrow x = 4$

$x = 4$ and $y = 8$

44. $f(x) = 12x + 3 \Rightarrow f(2) = (12 * 2) + 3 = 27$

45. $g(x) = 10x - 30 \Rightarrow g(3) = (10 * 3) - 30 = 0$

46. $\sqrt{x + 11} + 2 = 0 \Rightarrow \sqrt{x + 11} = -2 \Rightarrow$ unavailble to slove.

47. $x^3 + 27 = 0 \Rightarrow x = -3$

48. Slope of given line: $m = 4/3 \Rightarrow$ slope of perpendicular line: $-\frac{1}{m} = -3/4$

49. $2x + 3y = 11 \Rightarrow 3y = 11 - 2x \Rightarrow y = 11/3 - 2/3x$
slope of given line: $= -2/3 \Rightarrow$ slope of parallel line$= -2/3$

50. Expand $(x + 2)^3$

$= (x + 2)(x + 2)(x + 2) = (x^2 + 4x + 4)(x + 2)$

$= x^3 + 2x^2 + 4x^2 + 8x + 4x + 8$

$= x^3 + 6x^2 + 12x + 8$

51. Expand $\frac{a}{2}(6a + 4b - 3) = 3a^2 + 2ab - 3/2a$

52. $|x + 1| = 2 \Rightarrow x + 1 = 2$ or $x + 1 = -2$
$x = 1$ or $x = -3$

53. $|x - 4| = 1 \Rightarrow x - 4 = 1$ or $x - 4 = -1$
$x = 5$ or $x = 3$

54. $0.0000045 = 4.5 \cdot (10^{-6})$

55. $4,500,000 = 4.5 \cdot (1,000,000) = 4.5 \cdot (10^6)$

56. $(x - 1)|\overline{x^3 - 3x^2 + x + 1}$
$= x^2 - 2x - 1$

57. $(x-1)|\overline{x^3+3x^2-4}$
$= x^2+4x+4$ hint:$(x-1)|\overline{x^3+3x^2+0x-4}$

58. Solve for j in $7i+2k-3j=0$
$-3j=-7i-2k \Rightarrow j=(7/3)i+(2/3)k$

59. Solve for σ in $8\rho+2\theta-3\sigma=0$
$\sigma=(8/3)\rho+(2/3)\theta$

60. $\frac{x-2}{2-x}=\frac{x-2}{-1\cdot(x-2)}=\frac{1}{-1}\frac{x-2}{x-2}=-1\cdot 1=-1$

61. $\frac{-x+8}{x-8}=-1$

62. $(5/2)^2+(36/9)^{1/2}$

$=(5^2)/(2^2)+(36^{1/2})/(9^{1/2})$

$=25/4+16/3=33/4$

63. $(\frac{3}{4})^3+(\frac{27}{8})^{\frac{1}{3}}=123/64$

64. $x^{-2}\cdot x^3=(\frac{1}{x^2})\cdot x^3=\frac{x^3}{x^2}=x$

65. $x^{-4}\cdot x^6=x^2$

Third part.

66. a)$f(x)=\sqrt{10-2x} \Rightarrow 10x-2\geq 0 \Rightarrow -2x\geq -10 \Rightarrow x\leq 5$

b)$g(x)=\frac{1}{3x+2} \Rightarrow x\in \mathbf{R}$ and $x\neq -2/3$

67. $\sin(x)=1 \Rightarrow x=\sin^{-1}(1)=\pi/2 \Rightarrow x=\pi/2+2n\pi$, where n is any integer

68. $\cos(x)=\sqrt{3}/2 \Rightarrow x=\pi/6+2n\pi$, where n is any integer

69. $\sin^2\theta - \cos^2\theta = 1$ by Pythagorean theorem

$1 = \sin^2\theta + \cos^2\theta$ from unit circle

Thus, $\sin^2\theta - cos^2\theta = \sin^2\theta + \cos^2\theta$

$0 = 2\cos^2\theta \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2 + n\pi$

70. $\sin^2\theta - \cos^2\theta = -1 \Rightarrow \theta = \pi + n\pi$

71. $2\sec^2\theta - 4 = 0 \Rightarrow 2\sec^2\theta = 4 \Rightarrow \sec^2\theta = 2$

$\Rightarrow \frac{1}{\cos^\theta} = 2 \Rightarrow \cos^2\theta = 1/2 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} = \sqrt{2}/2$

$\theta = \pi/4 + 2n\pi$

72. $\sqrt{3}\csc\theta = 2 \Rightarrow \theta = \pi/3 + 2n\pi$ or $2\pi/3 + 2n\pi$

73. $\sin^2 x + 4\sin x + 4 = 0$

Let $u = \sin x$, then $u^2 + 4u + 4 = 0 = (u+2)^2$

$u + 2 = 0 \Rightarrow u = -2 \Rightarrow \sin x = -2 \Rightarrow x = \sin^{-1}(-2)$

74. $\cos^2 x - \cos x - 1 = 0$

$\cos x_{1,2} = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2} = \frac{1\pm\sqrt{5}}{2}$

$x_{1,2} = \cos^{-1}(\frac{1\pm\sqrt{5}}{2})$

75. $y = \sin x$ and $y = \cos x$.

Intersection point is where $\sin x = \cos x$. This occurs at $x = \pi/4$ and $5\pi/4$

76. $y = \csc x$ and $y = \sec x$. This occurs at $x = \pi/4$ and $5\pi/4$

77. graph 1

78. graph 2

Radians and Degrees

79. Convert.
$2\pi/9$ radians $=(2 * 180)/9 = 40$ degrees

80. Convert.
540 degrees $=3\pi$ radians

81. From $y = x^2$ to $y = (x-4)^2 \Rightarrow$ translated 4 units to the right.

82. From $y = (x-2)^2$ to $y = (x-3)^2 + 2 \Rightarrow$ translated 1 unit right and 2 units up.

83. $\sin 2\theta = \sin(\theta + \theta) = \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 2\sin\theta\cos\theta$

84. $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

85. graph 3
$3\pi/4$ radians in the unit circle.

86. graph 4
$4\pi/3$ radians in the unit circle.

87. $\sin 315^\circ = \sin(270^\circ + 45^\circ)$

$= \sin(270^\circ)\cdot\cos(45^\circ) + \cos(270^\circ)\cdot\sin(45^\circ) = -1\cdot(\sqrt{2}/2) + 0\cdot(\sqrt{2}/2) = \sqrt{3}/2$

88. $\cos 540^\circ = -1$

89. In which quadrants are the signs of cosine and cosecant the same?
Since $\csc\theta = 1/\sin\theta$, the sign of sin is the same for csc everywhere.
Thus we find the signs of sin and cos are the same in I and III.

90. In which quadrants are the signs of cosine and cosecant the same?
All of them

91. Vector: If $u = (3, 2)$ and $v = (1, 6)$, find $|2u - v|$

$2u - v =< 6, 4 > - < 1, 6 >=< 5, -2 >$

Thus $|2u - v| = \sqrt{5^2 + (-2)^2} = \sqrt{29}$

92. If $u =< 1, 3 >$ and $v =< 3, 0 >$, find $|u + v|$
Thus 5

93. graph 5

94. graph 6

95. What is the range of the cosine function.
Ans: All real numbers between -1 and 1, inclusive.

96. What is the range od the sine function.
Ans: All real numbers.

Trig Identities

97. Simplify.

$\frac{\sin 2x}{1-\sin^2 x} = \frac{\sin 2x}{\cos^2 x}$

since $\sin 2x = 2 \sin x \cos x$,

$\frac{sin2x}{\cos^2 x} = \frac{2 \sin x \cos x}{\cos^2 x} = \frac{2 \sin x}{cosx} = 2 \tan x$

98. $\cot 45^\circ = 1$

99. What is the period of $\sin x$
Ans: 2π

100. $f(2) = 3 \cdot 2 + 2 = 8$, $g(1) = 7 \cdot 1 = 7$, $k(3) = 3^2 = 9$
$f(2) + g(1) - k(3) = 8 + 7 - 9 = 6$